OCR
RECOGNISING ACHIEVEMENT

**OXFORD CAMBRIDGE AND RSA EXAMINATIONS**

**Advanced Subsidiary General Certificate of Education**
**Advanced General Certificate of Education**

# MATHEMATICS 4724

Core Mathematics 4

Monday **12 JUNE 2006** Afternoon 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
List of Formulae (MF1)

**TIME** 1 hour 30 minutes

## INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

## INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

**This question paper consists of 3 printed pages and 1 blank page.**

**1** Find the gradient of the curve $4x^2 + 2xy + y^2 = 12$ at the point $(1, 2)$. [4]

**2** **(i)** Expand $(1 - 3x)^{-2}$ in ascending powers of $x$, up to and including the term in $x^2$. [3]

**(ii)** Find the coefficient of $x^2$ in the expansion of $\dfrac{(1 + 2x)^2}{(1 - 3x)^2}$ in ascending powers of $x$. [4]

**3** **(i)** Express $\dfrac{3 - 2x}{x(3 - x)}$ in partial fractions. [3]

**(ii)** Show that $\displaystyle\int_1^2 \frac{3 - 2x}{x(3 - x)}\,dx = 0$. [4]

**(iii)** What does the result of part **(ii)** indicate about the graph of $y = \dfrac{3 - 2x}{x(3 - x)}$ between $x = 1$ and $x = 2$? [1]

**4** The position vectors of three points $A$, $B$ and $C$ relative to an origin $O$ are given respectively by

$$\overrightarrow{OA} = 7\mathbf{i} + 3\mathbf{j} - 3\mathbf{k},$$
$$\overrightarrow{OB} = 4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}$$
$$\text{and} \quad \overrightarrow{OC} = 5\mathbf{i} + 4\mathbf{j} - 5\mathbf{k}.$$

**(i)** Find the angle between $AB$ and $AC$. [6]

**(ii)** Find the area of triangle $ABC$. [2]

**5** A forest is burning so that, $t$ hours after the start of the fire, the area burnt is $A$ hectares. It is given that, at any instant, the rate at which this area is increasing is proportional to $A^2$.

**(i)** Write down a differential equation which models this situation. [2]

**(ii)** After 1 hour, 1000 hectares have been burnt; after 2 hours, 2000 hectares have been burnt. Find after how many hours 3000 hectares have been burnt. [6]

**6** **(i)** Show that the substitution $u = e^x + 1$ transforms $\displaystyle\int \frac{e^{2x}}{e^x + 1}\,dx$ to $\displaystyle\int \frac{u - 1}{u}\,du$. [3]

**(ii)** Hence show that $\displaystyle\int_0^1 \frac{e^{2x}}{e^x + 1}\,dx = e - 1 - \ln\left(\frac{e + 1}{2}\right)$. [5]

**7** Two lines have vector equations

$$\mathbf{r} = \mathbf{i} - 2\mathbf{j} + 4\mathbf{k} + \lambda(3\mathbf{i} + \mathbf{j} + a\mathbf{k}) \qquad \text{and} \qquad \mathbf{r} = -8\mathbf{i} + 2\mathbf{j} + 3\mathbf{k} + \mu(\mathbf{i} - 2\mathbf{j} - \mathbf{k}),$$

where $a$ is a constant.

**(i)** Given that the lines are skew, find the value that $a$ cannot take. [6]

**(ii)** Given instead that the lines intersect, find the point of intersection. [2]

**8** **(i)** Show that $\int \cos^2 6x \, dx = \frac{1}{2}x + \frac{1}{24}\sin 12x + c$. [3]

**(ii)** Hence find the exact value of $\int_0^{\frac{1}{12}\pi} x\cos^2 6x \, dx$. [6]

**9** A curve is given parametrically by the equations

$$x = 4\cos t, \qquad y = 3\sin t,$$

where $0 \leqslant t \leqslant \frac{1}{2}\pi$.

**(i)** Find $\frac{dy}{dx}$ in terms of $t$. [3]

**(ii)** Show that the equation of the tangent at the point $P$, where $t = p$, is

$$3x\cos p + 4y\sin p = 12.$$ [3]

**(iii)** The tangent at $P$ meets the $x$-axis at $R$ and the $y$-axis at $S$. $O$ is the origin. Show that the area of triangle $ORS$ is $\frac{12}{\sin 2p}$. [3]

**(iv)** Write down the least possible value of the area of triangle $ORS$, and give the corresponding value of $p$. [3]

| | | | | |
|---|---|---|---|---|
| **1** | | $\frac{d}{dx}(xy) = x\frac{dy}{dx} + y$ | B1 | s.o.i. e.g. $2x\frac{dy}{dx} + y$ |
| | | $\frac{d}{dx}(y^2) = 2y\frac{dy}{dx}$ | B1 | |
| | | Substitute (1,2) into their differentiated equation and attempt to solve for $\frac{dy}{dx}$. [Allow subst of (2,1)] | M1 dep at least 1 x B1 | Or attempt to solve their diff equation for $\frac{dy}{dx}$ and then substitute (1,2) |
| | | $\frac{dy}{dx} = -2$ | A1 **4** | |
| **2** | **(i)** | $1 + (-2)(-3x) + \frac{(-2)(-3)}{1.2}(-3x)^2$ (+ … ignore) | M1 | State or imply; accept $-3x^2$ & $-9x^2$ |
| | | $= 1 + 6x$ | **B**1 | Correct first 2 terms |
| | | $\ldots + 27x^2$ | A1 **3** | Correct third term |
| | **(ii)** | $(1+2x)^2(1-3x)^{-2}$ | M1 | For changing into suitable form, seen/implied |
| | | Attempt to expand $(1+2x)^2$ & select (at least) 2 relevant products and add | M1 | Selection may be after multiplying out |
| | | 55 (Accept $55x^2$) | A2√ **4** | If (i) is $a + bx + cx^2$, f.t. $4(a+b)+c$ |
| | | SR 1 For expansion of $(1+2x)^2$ with 1 error, A1√ | | |
| | | SR 2 For expansion of $(1+2x)^2$ & > 1 error, A0 | | |
| | | **Alternative Method** | | |
| | | For correct method idea of long division | M1 | |
| | | 1 ….. $+10x$ ….. $+55x^2$ | A1,A1,A1**(4)** | |
| **3** | **(i)** | $\frac{A}{x} + \frac{B}{3-x}$ & c-u rule or $A(3-x) + Bx \equiv 3 - 2x$ | M1 | Correct format + suitable method |
| | | $\frac{1}{x}$ | A1 | seen in (i) or (ii) |
| | | $-\frac{1}{3-x}$ | A1 **3** | ditto; $\frac{1}{x} - \frac{1}{3-x}$ scores 3 immediately |
| | **(ii)** | $\int \frac{1}{x}(dx) = \ln x$ or $\ln\|x\|$ | B1 | |
| | | $\int \frac{1}{3-x}(dx) = -\ln(3-x)$ or $-\ln\|3-x\|$ | B1 | Check sign carefully; do not allow $\ln(x-3)$ |
| | | Correct method idea of substitution of limits | M1 | Dep on an attempt at integrating |
| | | ln 2 (+ ln 1 − ln 1) − ln 2 = 0 | A1 **4** | Clearly seen; WWW **AG** |
| | | **Alternative Method** | | |
| | | If ignoring PFs, ln $x$(3 − $x$) immediately | B2 | $\ln x(x-3) \to 0$ |
| | | As before | M1,A1 **(4)** | |
| | **(iii)** | Suitable statement or clear implication e.g. Equal amounts (of area) above and below (axis) or graph crosses axis or there's a root (Be lenient ) | B1 **1** | |

| | | |
|---|---|---|
| **4** **(i)** Working out **b** – **a** or **a** – **b** or **c** – **a** or **a** – **c** | M1 ) | Irrespective of label |
| = ± (– 3**i** – **j** – **k**) or ± (– 2**i** + **j** – 2**k**) | A1 ) | If not scored ,these 1st 3 marks can be |
| Method for finding magnitude of <u>any</u> vector | M1 ) | awarded in part (ii) |
| Method for finding scalar product of <u>any</u> 2 vectors | M1 | |
| Using $\cos\theta = \frac{a.b}{\|a\|\|b\|}$ AEF for <u>any</u> 2 vectors | M1 | |
| [<u>Alternative cosine rule method</u> $\|\overrightarrow{BC}\| = \sqrt{6}$ | B1 | |
| Cosine rule used | M1 | 'Recognisable' form |
| $45.3°, 0.79(0), \frac{\pi}{3.97}$ (45.289378, 0.7904487) | A1 **6** | Do not accept supplement (134.7 etc) |
| **(ii)** Use of $\frac{1}{2}\|\overrightarrow{AB}\|\|\overrightarrow{AC}\|\sin\theta$ | M1 | Accept $\left\|\frac{1}{2}\overrightarrow{AB} \times \overrightarrow{AC}\right\|$ |
| 3.54 (3.5355) or $\frac{5\sqrt{2}}{2}$ | A1 **2** | Accept from correct supp (134.7 etc) |
| **5** **(i)** $\frac{dA}{dt}$ or $kA^2$ seen | M1 | |
| $\frac{dA}{dt} = kA^2$ | A1 **2** | |
| **(ii)** Separate variables + attempt to integrate | *M1 | Accept if based on $\frac{dA}{dt} = kA^2$ or $A^2$ |
| $-\frac{1}{A} = kt + c$ or $-\frac{1}{kA} = t + c$ or $-\frac{1}{A} = t + c$ | A1 | |
| Subst one of (0,0),(1,1000) or (2,2000) into eqn. | dep*M1 | Equation must contain $k$ and/or $c$ |
| Subst another of (0,),(1,1000) or (2,2000) into eqn | dep*M1 | This equation must contain $k$ <u>and</u> $c$ |
| Substitute $A = 3000$ into eqn with $k$ and $c$ subst | dep*M1 | |
| $t = \frac{7}{3}$ ISW | A1 **6** | Accept 2.33, 2h 20 m |
| **6** **(i)** Attempt to connect $du$ and $dx$ e.g. $\frac{du}{dx} = e^x$ | M1 | But not $du = dx$ |
| Use of $e^{2x} = (e^x)^2$ or $(u-1)^2$ s.o.i. | A1 | |
| Simplification to $\int \frac{u-1}{u}(du)$ WWW | A1 **3** | **AG** |
| **(ii)** Change $\frac{u-1}{u}$ to $1 - \frac{1}{u}$ or use parts | M1 | If parts, may be twice if $\int \ln x \, dx$ is involved |
| $\int \frac{1}{u} du = \ln u$ | A1 | Seen anywhere in this part |
| <u>Either</u> attempt to change limits <u>or</u> resubstitute | M1 (indep) | Expect new limits e+1 & 2 |
| Show as $e + 1 - \ln(e+1) - \{2 \text{ or } (1+1)\} + \ln 2$ | A1 | |
| WWW show final result as $e - 1 - \ln\left(\frac{e+1}{2}\right)$ | A1 **5** | **AG** |
| **7** **(i)** Produce at least 2 of the 3 relevant eqns in λ and μ | M1 | e.g. 1 + 3λ = –8 + μ, –2 + λ = 2 –2 μ |
| Solve the 2 eqns in λ & μ as far as λ = … or μ = … | M1 | |
| 1st solution: λ = –2 or μ = 3 | A1 | |
| 2nd solution: μ = 3 or λ = –2 f.t. | A1√ | |
| Substitute their λ and μ into 3rd eqn and find '$a$' | M1 | |
| Obtain $a = 2$ <u>& clearly state that $a$ cannot be 2</u> | A1 **6** | |
| **(ii)** Subst their λ or μ (& poss $a$) into either line eqn | M1 | |
| Point of intersection is $-5\,\mathbf{i} - 4\,\mathbf{j}$ | A1 **2** | Accept any format <u>No f.t. here</u> |
| **N.B.** In this question, award marks irrespective of labelling of parts | | |

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| 8 | (i) | Integration method | | |
| | | Attempt to change $\cos^2 6x$ into $f(\cos 12x)$ | M1 | |
| | | $\cos^2 6x = \frac{1}{2}(1+\cos 12x)$ | A1 | with $\cos^2 6x$ as the subject of the formula |
| | | $\int = \frac{1}{2}x + \frac{1}{24}\sin 12x + c$ | A1 | **AG** Accept $\frac{1}{2}\left(x+\frac{1}{12}\sin 12x\right)$ |
| | | Differentiation method | | |
| | | Differentiate RHS producing $\frac{1}{2}+\frac{1}{2}\cos 12x$ ---(E) | B1 | |
| | | Attempt to change $\cos 12x$ into $f(\cos 6x)$ | M1 | Accept $+/-2\cos^2 6x + /-1$ |
| | | Simplify (E) WWW to $\cos^2 6x$ + satis finish | A1 **3** | |
| | (ii) | Parts with $u = x, \mathrm{d}v = \cos^2 6x$ | *M1 | |
| | | $x\left(\frac{1}{2}x+\frac{1}{24}\sin 12x\right)-\int\left(\frac{1}{2}x+\frac{1}{24}\sin 12x\right)\mathrm{d}x$ | A1 | Correct expression only |
| | | $\int \sin 12x\,\mathrm{d}x = -\frac{1}{12}\cos 12x$ | B1 | Clear indication somewhere in this part |
| | | Correct use of limits to whole integral | dep*M1 | Accept ( ) ( −0 ) |
| | | $\frac{\pi^2}{288}-\frac{\pi^2}{576}-\frac{1}{288}-\frac{1}{288}$ | A1 | AE unsimp exp. Accept 12x24,sin $\pi$ here |
| | | $\frac{\pi^2}{576}-\frac{1}{144}$ | +A1 **6** | Tolerate e.g. $\frac{2}{288}$ here |
| | | S.R. If final marks are A0 + A0, allow SR A1 for | | 0.01/0.010/0.0101/0.0102/0.0101902 |
| 9 | (i) | $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\frac{\mathrm{d}y}{\mathrm{d}t}}{\frac{\mathrm{d}x}{\mathrm{d}t}}$ | M1 | Used, not just quoted |
| | | $\frac{\mathrm{d}x}{\mathrm{d}t} = -4\sin t$ or $\frac{\mathrm{d}y}{\mathrm{d}t} = 3\cos t$ | *B1 | |
| | | $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{3\cos t}{4\sin t}$ or $\frac{3\cos t}{-4\sin t}$ ISW | dep*A1 **3** | Also $\frac{-3\cos t}{4\sin t}$ provided B0 not awarded |
| | | SR: M1 for Cartesian eqn attempt + B1 for $\frac{\mathrm{d}}{\mathrm{d}x}\left(y^2\right) = 2y\frac{\mathrm{d}y}{\mathrm{d}x}$ | | + A1 as before(must be in terms of $t$) |
| | (ii) | $y - 3\sin p = \left(\text{their } \frac{\mathrm{d}y}{\mathrm{d}x}\right)(x - 4\cos p)$ | M1 | Accept $p$ or $t$ here |
| | | or $y = \left(\text{their } \frac{\mathrm{d}y}{\mathrm{d}x}\right)x + c$ & subst cords to find c | | Ditto |
| | | $4y\sin p - 12\sin^2 p = -3x\cos p + 12\cos^2 p$ | A1 | Correct equation cleared of fractions |
| | | or $c = \frac{12\sin^2 p + 12\cos^2 p}{4\sin p}$ | | |
| | | $3x\cos p + 4y\sin p = 12$ WWW | A1 **3** | **AG** Only $p$ here. Mixture earlier $\rightarrow$ A0 |
| | (iii) | Subst $x = 0$ and $y = 0$ separately in tangent eqn | M1 | to find $R$ & $S$ |
| | | Produce $\frac{3}{\sin p}$ and $\frac{4}{\cos p}$ | A1 | Accept $\frac{12}{4\sin p}$ and/or $\frac{12}{3\cos p}$ |
| | | Use $\Delta = \frac{1}{2}\left(\frac{3}{\sin p}.\frac{4}{\cos p}\right) = \frac{12}{\sin 2p}$ WWW | A1 **3** | **AG** |
| | (iv) | Least area = 12 | B1 | |
| | | $p = \frac{1}{4}\pi$ as final or only answer | B2 **3** | These B marks are independent. |
| | | S.R. 45 ° $\rightarrow$ B1 ; | | S.R. [ $-12$ and e.g. $-\pi/4 \rightarrow$ B1 ] |